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Green Theorems and Qualitative Properties of the Optical Flow

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Abstract

How can one compute qualitative properties of the optical flow, such as expansion or rotation, in a way which is robust and invariant to the position of the focus of expansion or the center of rotation? We suggest a particularly simple algorithm, well-suited to VLSI implementations, that exploits well-known relations between the integral and differential properties of vector fields and their linear behaviour near singularities.

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13. ABSTRACT *(Maximum 200 words)*

How can one compute qualitative properties of the optical flow, such as expansion or rotation, in a way which is robust and invariant to the position of the focus of expansion or the center of rotation? We suggest a particularly simple algorithm, well-suited to VLSI implementations, that exploits well-known relations between the integral and differential properties of vector fields and their linear behaviour near singularities.

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The optical flow is the field of apparent velocities derived from the time-dependent image. On the basis of computational arguments it has been suggested [15, 16] that measurements of the optical flow may be used not for a quantitative estimate of physical 3D velocities for which the optical flow is ill-suited, but rather for extracting qualitative properties, such as expansion or rotation. These qualitative properties relate directly to physically meaningful properties of the motion and of the 3D structure of the environment [4, 12]: for instance, expansion may signify a fast approach to a surface, that is an imminent crash, while detection of rotation and of its sign may be used to control egomotion. Detectors that could discriminate between translation, expansion and rotation of the optical flow could be quite useful for several visual tasks, such as, for instance, navigation [3, 5, 6, 2]. Such detectors should provide measurements of expansion, contraction and translation, in a way which is ideally robust against noise in the measurements and invariant to the position of the focus of expansion (or contraction) or of the center of rotation in the optical flow.

Consider for example the optical flow fields shown in fig.1. Suppose that local measurements of the optical flow are available (one measurement, say, for each of the arrows shown in the figure), as provided by elementary motion detectors. How can one estimate expansion with the property of invariance to the position of the focus? If the measurements of the flow were available everywhere one could compute at any given point the divergence of the field $\nabla \mathbf{u}(x,y)$ which is a differential measure of the local expansion ($\nabla \mathbf{u}(x,y) = (\frac{\partial u_x(x,y)}{\partial x}) + (\frac{\partial u_y(x,y)}{\partial y})$). For a linear field (i.e. $\mathbf{u}(\mathbf{x}) = A\mathbf{x}$), the divergence of $\mathbf{u}$ is the same everywhere (in the linear case the magnitude of the vectors is proportional to the distance from the focus of expansion or from the center of rotation). It follows that the average over an area (see the dashed circle in Fig 1A and 1B) of $\nabla \mathbf{u}(x,y)$ is the same for any position of the circle relative to the optical flow. In the case of linear fields, the integral of the divergence over an area has therefore *exactly* the desired invariance property. A structurally stable planar field can always be approximated around a singularity by a linear field. In practice the invariance property holds *approximately* in most cases within reasonably large distances of the singularity [12]. Derivatives such as the divergence, however, are numerically ill-conditioned (especially because the elementary measurements of the flow require themselves time and space derivatives – or the equivalent thereof, see for instance [9] – applied to

the image data, i.e. to the photoreceptors signals). In addition a divergence-based implementation would need a large set of dense measurements, which is unfeasible in practice. Fortunately, there exist theorems, called Green 's theorems, that show that the integral over a surface patch S of the divergence of a field $\mathbf{u}$ is equal to the integral along the patch boundary of the component of the field which is normal to the boundary $(\mathbf{u} \cdot \mathbf{n})$. In formula

$$\int_S \nabla \mathbf{u}(x, y) dx dy = \int_C \mathbf{u} \cdot \mathbf{n} dl. \tag{1}$$

Thus, a detector that exploits Green theorem just needs to summate over, say, a circle (or any close contour) the normal component of the flow, measured along the circle. This sum is invariant to the position of the focus of expansion relative to the circle because of Green theorem (for a linear field). This argument justify the circuitry of figure 2a, in which elementary motion detectors measuring the radial component of optical flow are summated over the "receptive" field of the detector. In addition to the desired position invariance, this scheme is quite robust, since Green's theorem effectively eliminates noise-prone derivatives from the computation. Thus, a detector for expansion of the flow that is based on this scheme can be expected to be *quite robust under noisy* and sparse measurements of the flow.

Notice that a *precise* measurement of the normal component of the flow is not really necessary, since the precise definition of the optical flow is itself somewhat arbitrary [15]: it is sufficient that the estimate be qualitatively consistent with the values of the perspective 2D projection of the "true" 3D velocity field [14].

We have discussed so far schemes for detecting expansion. Similar arguments hold for rotation. The Green 's theorem relevant to this case is usually called Stokes' theorem and takes the form

$$\int_A \nabla \wedge \mathbf{u}(x, y) \cdot d\mathbf{A} = \int_C \mathbf{u} \cdot d\mathbf{r} \tag{2}$$

which says that the total flux of the differential measure of "rotationality" of the field $\nabla \wedge \mathbf{u}$ across the surface patch A is equal to the integral along the boundary C of the surface patch of the component of the field which is tangential to the boundary. Figure 2b shows the corresponding circuitry for a detector that is sensitive to rotation of the field. Again the output of the detector is invariant to the position of the center of rotation (exactly for linear fields) and its performance is robust since derivatives are not required.

Our preliminary numerical experiments, some of which are shown in fig.3, confirm that the algorithms we propose are position invariant for linear fields and robust, in the sense that they provide good estimates of rotation and/or expansion with noisy data and very few inputs (from elementary motion detectors).

Thus the advantages of the basic scheme shown in fig. 2a and 2B are three:

1) it is invariant to the position of the focus of expansion (or contraction) or to the position of the center of rotation for linear fields ;

2) it avoids the computation of derivatives of the flow that are numerically unstable and noise sensitive;

3) it uses only one component of the flow (the radial one or the tangential one to the patch boundary, depending on what qualitative property the unit detects).

The circuitry of Fig 2a and 2B represents a simple and robust – though obvious – algorithm for machine vision applications. It suggests directly VLSI circuit implementations for "crash" and rotation detectors. In addition, the same basic scheme is suggestive of properties shown by cortical cells in aerea MST and we had in fact conjectured that MST cells may in fact exploit Green's theorem ([8]). Recent evidence however does not seem to confirm this view (Orban et al., 1991 in preparation).

In this paper we have not discussed how to measure the radial or the tangential component of the optical flow. As we mentioned earlier, the specific type of elementary motion detectors that are used to provide the estimate of the normal component of the flow is probably not critical. Radially oriented (for expansion and contraction), two input elementary motion detectors such as the correlation model [10, 11, 7, 13] – or approximations of it are likely to be adequate. The critical property is that they should measure motion with the correct sign.

Let us briefly discuss the connection between the proposed methods and previous work. Obviously, it was not our goal to develop algorithms for the localization of the focus of expansion in the optical flow (like in ref. [3], [5] and [2], for example). Instead, our use of Green's theorems is a simple idea with the restricted objective of providing robust estimates of qualitative properties of optical flow, such as expansion and rotation, independently of the position of the singularities of the flow. Recently, different methods for obstacle avoidance and image segmentation have been proposed, in which the

estimate of qualitative properties of optical flow is obtained either through spatial derivatives of optical flow [6] or least mean square techniques [1]. In order to obtain meaningful results precise estimates of these properties are essential. We believe that the simple algorithms discussed in this paper represent an alternative and appealing scheme for the computation of those qualitative properties in the presence of noise.

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Figure captions

Fig. 1:

Optical flow fields corresponding to expansion (a) and clockwise rotation (b). The arrows indicate local direction and magnitude of the optical flow. The dotted circle indicates the part of the flow seen by the detector proposed in this paper.

Fig. 2:

(A) shows the scheme that we propose for detecting expansion. The detector summates inputs from elementary motion detectors that measure an approximation of the radial component of the optical flow. Notice that in the case of linear expanding fields, $\nabla \mathbf{u}$ is an estimate of the inverse of the time to crash. (B) is the equivalent model for rotation. The arguments in the text show that these schemes are largely invariant to the position of the circle – the summation area – relative to the focus of expansion or the center of rotation.

Fig. 3:

(A) shows the response of the detector tuned for expansion (see Fig. 2a) for the three different positions of the detecors visual field shown in (B). For each position the output of the detector is shown for an expanding field (the situation depicted in Fig. 3B), for a translating field and for a rotating field.

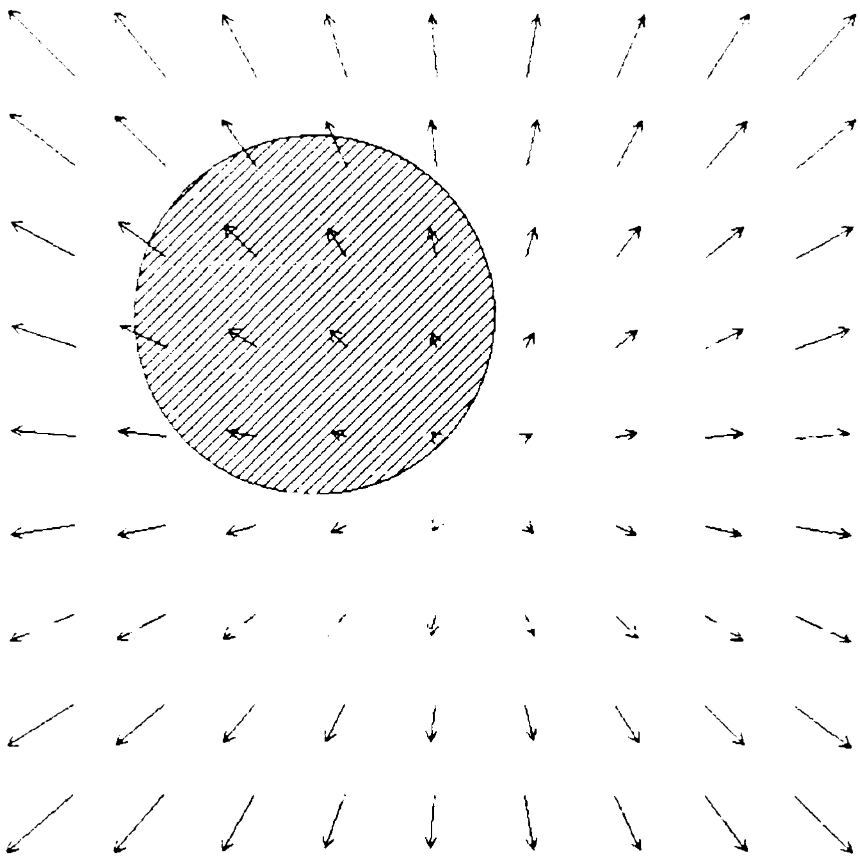

FIGURE 1A

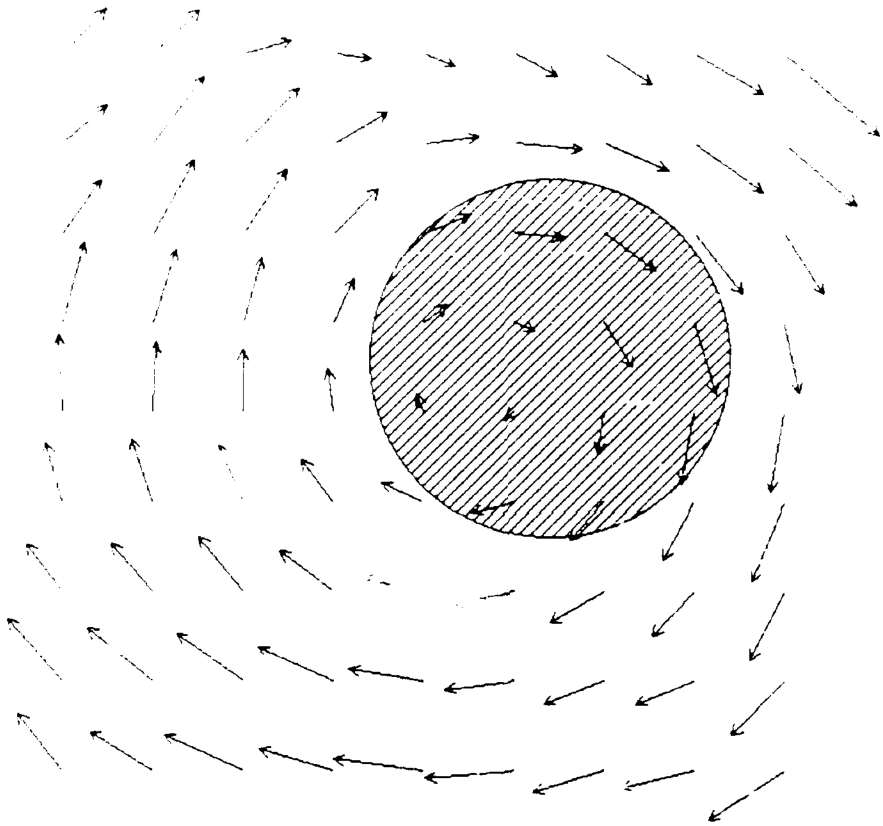

FIGURE 1B

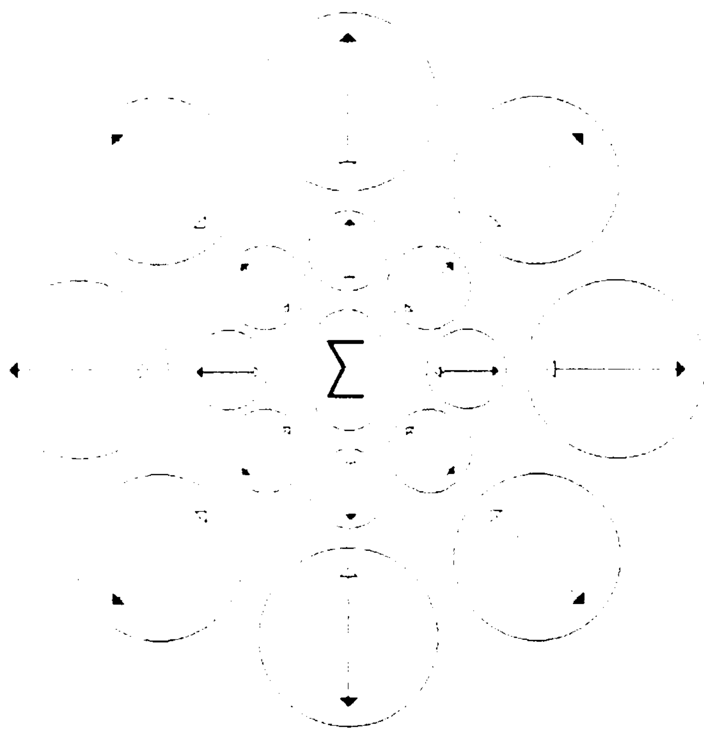

FIGURE 2A

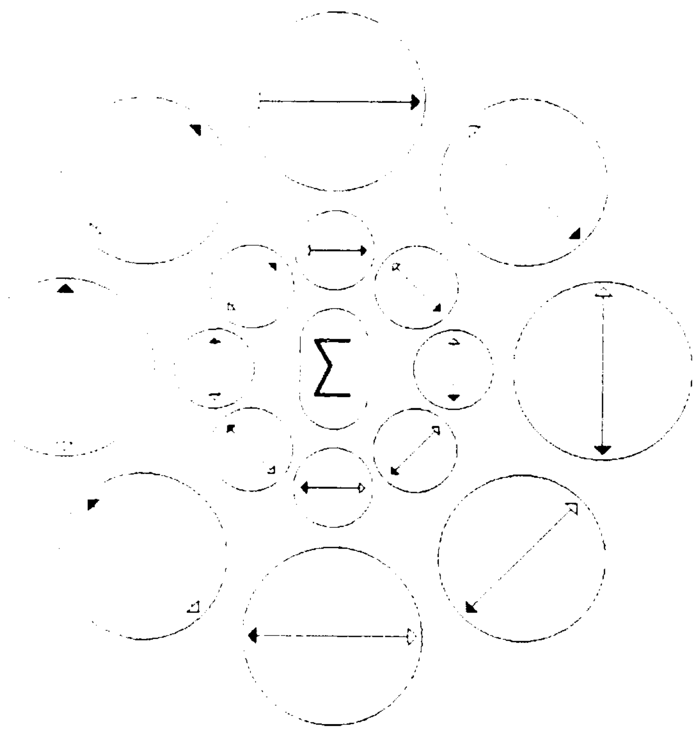

FIGURE 2B

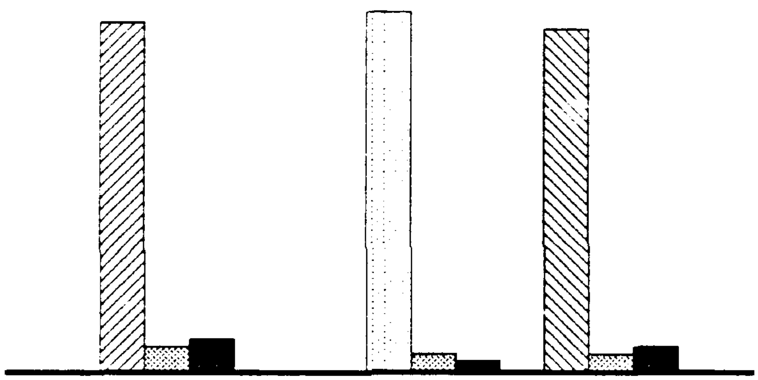

FIGURE 3A

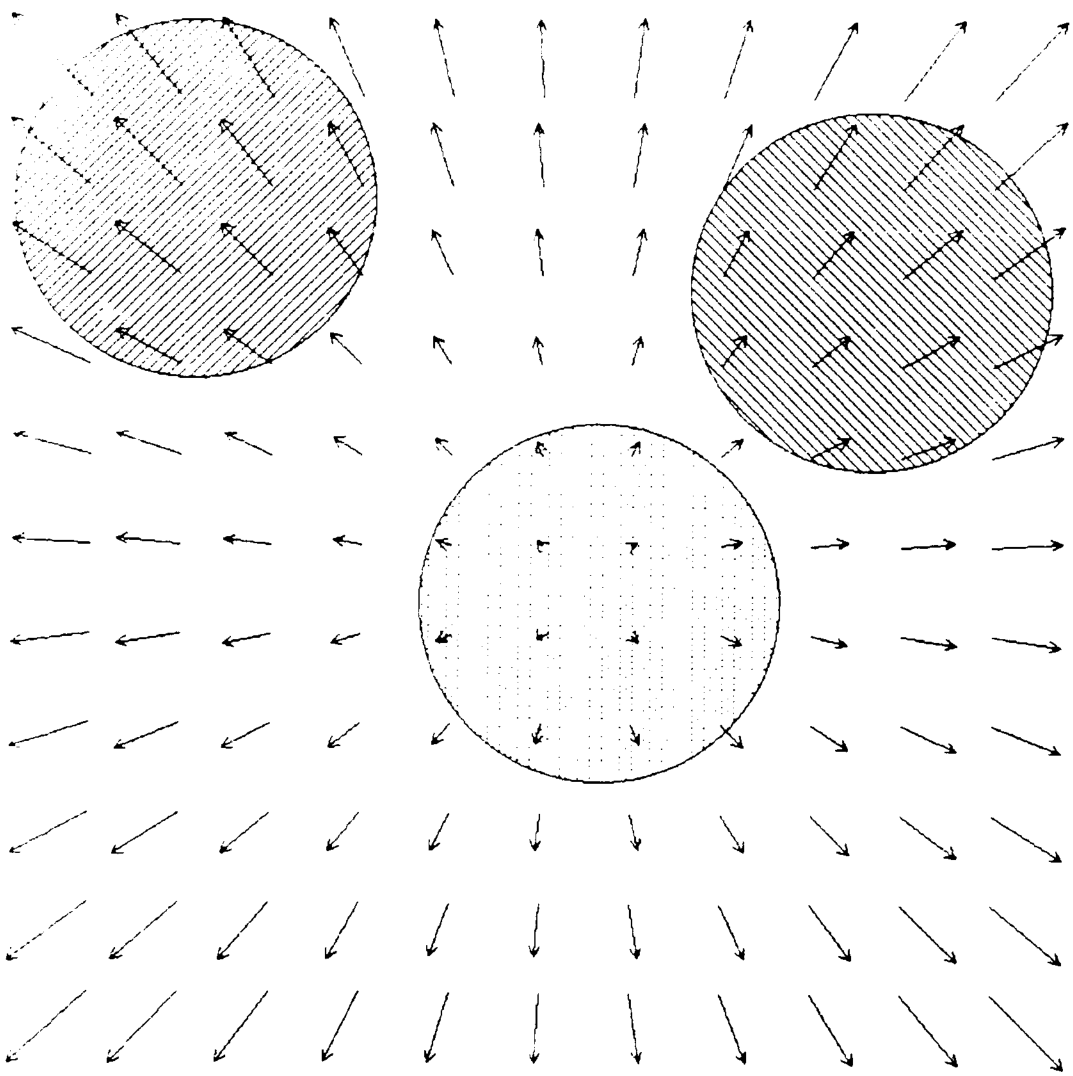

FIGURE 3B